The theoretical distance of Neptune (in column A) appears to be an *exact* mean proportional between Mercury's theoretical distance and the modulus of light. That modulus, according to this determination, is 476,198 $\times$ Sun's radius; according to Struve's value of the constant of aberration, it is $(475{,}969.23 \pm 258.45) \times$ solar radius.

The theoretical series is symmetrical, in having three terms in arithmetical progression at either extremity. This analogy is more nearly carried out in the actual positions of the three exterior planets, which have been regarded as exceptional, than in those of the three interior planets, which have been considered normal.

The theoretical positions of Mercury and Venus, are at centres of direct and reverse oscillation between Earth and $\frac{\pi}{32}$; those of Uranus and Saturn, at similar centres between Sun and Neptune.

The successive doubling of the differences, places each of the theoretical intermediate planets at a centre of oscillation between the next inferior and the next superior planet.

The deviations from theoretical positions, in consequence of mutual planetary disturbances, distribute the planets in various symmetrical ways.

The exponents of the divisor, E, are arranged symmetrically in pairs. ($\frac{2}{3}$, 0; $\frac{1}{3}$, 1; $\frac{1}{3}$, 1; $\frac{2}{3}$, 0.)

The four central planets are grouped, by their divisors, in alternate pairs; Earth, Jupiter; Mars, Saturn.

The four terminal planets are similarly grouped; Neptune, Venus; Uranus, Mercury.

If the division by π be thrice repeated, below the theoretical position of Mercury, we obtain, very nearly, $(\pi - 1) \times$ solar radius, or the diameter of the circle described by the centre of gravity of Sun and Jupiter.

ROTATION OF THE SUN AND THE INTRA-ASTEROIDAL PLANETS.

By Pliny Earle Chase.

(*Read before the American Philosophical Society, March 7th,* 1873.)

The well known tendency to synchronism in concurrent vibrations, has encouraged me to look for some significant harmony between the lengths of solar and planetary days and times of fall to the centre of the system.

The middle term in my series of alternate planetary distances, differs from the others in having a multiple significance, representing, as it does, a mean position in the asteroidal belt and the orbital major axis of Mars. It has also simple relationships to the distances and rotation-times of the intra-asteroidal planets, which serve to connect the diurnal with the annual motions, and both with the equilibrating forces of the Sun.

Since the velocities of falling and oscillating bodies are proportioned to the times of actual or virtual fall, rotary and orbital velocities can be compared by a simple and natural hypothesis. Let

ρ = modulus of rotation, or distance from Sun through which a planet would fall in τ^f. $\rho^{\frac{3}{2}} \propto \sqrt{\frac{}{32}} \times \tau^f$

τ^f = time of fall which will satisfy the proportion.

$$\tau^f : \tau^r :: V : v$$

τ^r = length of day.

V = orbital velocity.

v = equatorial velocity of rotation $= \frac{2 \pi r}{\tau^r}$

$$= \frac{d}{\delta} = \frac{\pi^*}{\sqrt{8}}$$

Substituting for v its value, in the above proportion, and reducing, we obtain the equation

$$\tau^r = \left[\frac{2 \pi r \tau^f}{V}\right]^{\frac{1}{2}}$$

The Earth being the controlling planet of the intra-asteroidal system, it is not surprising that its radius vector and the factor of equilibrium, ,

both enter into the values of ρ. 2 $^4 = \frac{\pi^4}{32}$, and $\frac{\pi^4}{32} \times$ Earth's mean radius vector = major axis of Mars, which in its turn = mean radius vector — modulus of rotation of Mercury; Earth's ρ = its perihelion distance : the moduli of rotation for Venus and Mars are, respectively, and ^{2} Earth's modulus; Sun's modulus = mean distance of Uranus = distance of Neptune's radial centre of oscillation. The accordance of these theoretical values with the actual values, is shown in the following table :

MODULI AND TIMES OF ROTATION.

	ρ	Theoretical τ^r.	Observed τ^r.
Mercury	2.653	24.321	24.091
Venus	1.093	23.342	23.357
Earth	.983	23.972	23.934
Mars	1.213	24.537	24.622
Sun	19.183	603.138	604.007

The solar modulus serves to connect the synchronism of light-oscillation at Neptune and planetary oscillation at Sun with the light-velocity which would be communicated by the action of solar-superficial gravity in a half-rotation.

The modulus of rotation for Mercury (2.653 ; cfr. Juno, 2.671 ; $\frac{1}{2}$ Jupiter, 2.601 ; $\frac{8}{3}$ Earth, 2.667), may, perhaps, indicate a fall from the middle of the asteroidal belt, since the time when Mars and Venus were thrown off from the nebulous earth. Or, inasmuch as ^{3} is the aphelion distance

* See foregoing note on the "Estimation of Solar Mass and Distance."

of a planet with a major axis equivalent to Mercury's modulus, and eccentricity equivalent to the mean of the eccentricities of the other interior planets, Mercury and Mars may have condensed simultaneously from an intra-asteroidal ring, of a thickness corresponding to the difference between their rotation-moduli. $\angle^4$ would then indicate the primary nucleus of the first intra-asteroidal nebulous ring, or the mean distance of Mars; $\triangle^3$, the perihelion of Mars; $\triangle^2$, $\triangle^1$, $\triangle^0$, the moduli of rotation of Mars, Venus, and Earth.

PLANETARY RELATIONS TO THE SUN-SPOT PERIOD.

By Pliny Earle Chase.

(*Read before the American Philosophical Society, March 7th*, 1873.)

Stockwell's discussion of Secular Variations has furnished an unexpected confirmation of my suggested accordance between Jupiter's mean perihelion distance, the planetary centre of gyration, and the radius vector of the disturbing force which occasions the mean sun-spot cycle of 11.07 ± years.

In my previous paper (*ante*, xii, 410) I made the comparisons with the present eccentricity of Jupiter. If we take the *mean* eccentricity (.04316), Jupiter's mean perihelion is in the precise orbit of the disturbing force, provided the disturbance-period is 11.11 years. The factor of Jupiter's variation from Bode's Law $\left[(1.079)^{\frac{1}{3}}\right]$ is also the factor of the perturbation variation from the centre of planetary gyration (5.101 ÷ 1.0257 = 4.973).

Kirkwood has shown the approximate commensurability of the Wolfian cycle with 46 years of Mercury, 18 years of Venus, 11 of Earth, 6 of Mars, and 1 of Jupiter. I have introduced these five periods in the following table, together with (6) $\frac{3}{8}$ of Saturn, (7) $\frac{2}{15}$ of Uranus; (8) $\frac{1}{15}$ of Neptune; (9) $\frac{2}{3}$ year of the mean centre of inertia of Jupiter's aphelion and Saturn's perihelion; (10) $\frac{2}{3}$ do. Jupiter's perihelion and the aphelion of Uranus; (11) $\frac{2}{7}$ do. Saturn's aphelion and Uranus' perihelion; (12) $\frac{1}{5}$ do. Saturn's perihelion and Neptune's aphelion; (13) $\frac{1}{12}$ do. Uranus' aphelion and Neptune's perihelion.

APPROXIMATIONS TO THE WOLFIAN CYCLE.

1.	46	years of Mercury	4046.63
2.	18	" Venus	4044.60
3.	11	" Earth	4017.86
4.	6	" Mars	4121.86
5.	1	" Jupiter	4332.58
6.	$\frac{3}{8}$*	" Saturn	4034.71
7.	$\frac{2}{15}$	" Uranus	4091.78
8.	$\frac{1}{15}$	" Neptune	4008.45

* Earth's radius vector, divided by $\frac{3}{8}$ = Mercury's modulus of rotation.